Now we get the optimal soln. by following tables.

	1	2	3	4	5	6
1	1.5	1	1	$\cancel{0}$	$\boxed{0}$	3
2	$\boxed{0}$	0	0.5	0	1	2
3	$\cancel{0}$	0.5	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	1
4	0.5	1	$\boxed{0}$	1	1.5	1
5	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	10	2	$\cancel{0}$
6	2	2	2	1	2	$\boxed{0}$

	1	2	3	4	5	6
1	1.5	1	1	0	$\boxed{0}$	3
2	$\cancel{0}$	$\boxed{0}$	0.5	$\cancel{0}$	1	2
3	$\boxed{0}$	0.5	$\cancel{0}$	0	$\cancel{0}$	1
4	0.5	1	$\boxed{0}$	1	1.5	1
5	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	2	0
6	2	2	2	1	2	$\boxed{0}$

	1	2	3	4	5	6
1	1.5	1	1	$\boxed{0}$	0	3
2	$\boxed{0}$	$\cancel{0}$	0.5	$\cancel{0}$	1	2
3	$\cancel{0}$	0.5	$\boxed{\cancel{0}}$	$\cancel{0}$	$\boxed{0}$	1
4	0.5	1	$\boxed{0}$	1	1.5	1
5	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	2	$\boxed{\cancel{0}}$
6	2	2	2	1	2	$\boxed{0}$

	1	2	3	4	5	6
1	1.5	1	1	$\boxed{0}$	$\cancel{0}$	3
2	$\cancel{0}$	$\boxed{0}$	0.5	$\cancel{0}$	1	2
3	$\boxed{\cancel{0}}$	0.5	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	1
4	0.5	1	$\boxed{0}$	1	1.5	1
5	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	2	0
6	2	2	2	1	2	$\boxed{0}$

	1	2	3	4	5	6
1	1.5	1	1	$\cancel{0}$	$\boxed{0}$	3
2	$\cancel{0}$	$\cancel{0}$	0.5	$\boxed{0}$	1	2
3	$\boxed{0}$	0.5	$\cancel{0}$	$\cancel{0}$	0	1
4	0.5	1	$\boxed{0}$	1	1.5	1
5	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	2	0
6	2	2	2	1	2	$\boxed{0}$

	1	2	3	4	5	6
1	1.5	1	1	$\cancel{0}$	$\boxed{0}$	3
2	$\cancel{0}$	$\boxed{0}$	0.5	$\cancel{0}$	1	2
3	$\boxed{0}$	0.5	$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	1
4	0.5	1	$\boxed{0}$	1	1.5	1
5	0	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	2	0
6	2	2	2	1	2	$\boxed{0}$

eg 12.

Subordinates

Tasks	I	II	III	IV
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

How should be the task allocated one to a man, so as to minimize the total man hour.

Soln. Subtract the minimum element from each row,

	I	II	III	IV
A	0	18	9	3
B	9	24	0	22
C	23	4	3	0
D	9	16	14	0

Now subtract the minimum value from each column,

	I	II	III	IV
A	$\boxed{0}$	14	9	3
B	9	20	$\boxed{0}$	22
C	23	$\boxed{0}$	3	0
D	9	12	14	$\boxed{0}$

A	B	C	D	Task
I	III	II	IV	Subordinate
8	4	19	10 .	Manhour.

Total man hour $\Rightarrow 8+4+19+10=41$

Hungarian Method (Reduced Matrix Method)

eg:- Solve the following minimal assignment problem.

Jobs → / Man →	1	2	3	4
I	12	30	21	15
II	18	33	9	31
III	44	25	24	21
IV	23	30	28	14

→ Balanced Problem.

Solution:

Step-1 Subtracting the smallest element of each row from every element of the corresponding row, we get the following matrix.

	1	2	3	4
I	0	8	9	3
II	9	24	0	22
III	23	4	3	0
IV	9	16	14	0

Step-2 Subtracting smallest element of each Column from every element of the corresponding Column, we get the following matrix.

	1	2	3	4
I	0	4	9	3
II	9	20	0	22
III	23	0	3	0
IV	9	12	14	0

Step-3 Now we test whether it is possible to make an assignment using the zeroes.

Starting with row 1, we mark □ in the row containing only one zero and cross (x) the zero in the corresponding in which □ lies. Thus we get the following table.

$\boxed{0}$	4	9	3
9	20	$\boxed{0}$	22
23	0	3	$\cancel{0}$
9	12	14	$\boxed{0}$

→

$\boxed{0}$	4	9	3
9	20	$\boxed{0}$	22
23	$\boxed{0}$	3	$\cancel{0}$
9	12	14	$\boxed{0}$

→ Again starting with Column 1, we mark □ in the column containing only one unmarked zero in the above table and cross out (x) the zeroes in the corresponding row.

Since in the last table, every row and every column have one assigned, so we have the complete optimal zero assignment.

Job	I	II	III	IV
Man	1	3	2	4

which is the optimal sol.

Unbalanced Assignment problem:-

When the no of rows ≠ no of Column, then it is said to be unbalanced. For the solution of such problem we add the dummy rows or Columns to the given matrix to make it a square matrix. The costs in these dummy rows or columns are taken to be zero, Now the problem reduced to the balanced assignment problem and can be solved by assignment alogrithm.

Proof: We have,

$$z' = \sum_{i=1}^{m} \sum_{j=1}^{n} c'_{ij} x_{ij}$$

$$= \sum_{i=1}^{m} \sum_{j=1}^{n} (c_{ij} \pm a_i \pm b_j)\, x_{ij}$$

$$= \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \pm \sum_{i=1}^{m} \sum_{j=1}^{n} a_i x_{ij} \pm \sum_{i=1}^{m} \sum_{j=1}^{n} b_j x_{ij}$$

$$= z \pm \sum_{i=1}^{m} a_i \cdot \sum_{j=1}^{n} x_{ij} \pm \sum_{i=1}^{m} x_{ij} \cdot \sum_{j=1}^{n} b_j$$

$$= z \pm \sum_{i=1}^{m} a_i \cdot 1 + \sum_{j=1}^{n} \cdot 1$$

$$= z \pm \sum_{i=1}^{m} a_i + \sum_{j=1}^{n} b_j$$

Since $\sum_{i=1}^{m} a_i$, $\sum_{j=1}^{n} b_j$ are independent of x_{ij} it follows that z' is minimized when z is minimized.

Hence, $x_{ij} = X_{ij}$ which minimizes z also minimize z'.

Theorem 6.2 If all $c_{ij} \geq 0$ and there exist a solution

$x_{ij} = X_{ij}$ s.t $\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} = 0$,

then this solution is an optimal solution (i.e the solution minimizes z)

Proof: Since all $c_{ij} \geq 0$

$z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \cdot x_{ij}$ Can not be negative.

Thus its minimum value is 0, when $x_{ij} = X_{ij}$

Hence the solution $x_{ij} = X_{ij}$ for which $\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} = 0$ is an optimal soln.

Assignment Problem

Balanced Assignment Problem :- If the no of rows is equal to the no of Coloum than it is said to balanced assignment problem, if no(R) ≠ no(C) than the problem is unbalanced and we convert the unbalanced problem into balanced and solve by Hungarian method.